STRUCTURAL SIMULATION OF SUPERCOMPRESSIBLE MATERIALS

S.V. Shilko, S.V. Stelmakh, D.A. Chernous & Yu.M. Pleskatchevskii

Metal-Polymer Research Institute of National Academy of Sciences;
32a, Kirov Street 246050 Gomel, Belarus
e-mail: shilko_mpri@mail.ru

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Abstract

The work is devoted to study of supercompressibility effect, i.e. anomaly of elasticity, when Poisson’s ratio become negative.

The numerical results provide the conclusion that structural modelling of granular media, honeycomb structures and foams demonstrates the phenomena mentioned above. The new localization method of structural unity creation and controlling has been developed for elasitic and non-elastic properties prediction.

Introduction

The absolute majority of raw materials and artificial composites demonstrates nonlinearity of elastic behaviour [1] and some of them having unusial elasic properties, such as supercompressible isotropic foams [2]. The remarkable anomaly of deforming process may be produced when we have the specific combination of geometry and loading conditions of structural unity [3].

1. The localization method

The deformation properties of composite forming by conjoint bodies materials combination depend on interface mechanical characteristics. Particularly, the study of deforming process for granular solids interacting by presence of reversible frictional and adhesion forces includes the problem of free boundary, i.e. creation and evolution of zones with different boundary conditions on microcontacts (figure 1.1). When the additional loading in two-interface system (figure 1.2) occurs, the localization or controlling of free boundaries evolution is possible.

The size and location of contact area as a whole are apriory unknown in common case. For the microslip on granulars interfaces, initiated by bulk deformation, when static (in stick region) and kinetic (in slip region) friction are realized simultaneously the internal boundary between these specific zones should be determined too. The frictional bond rupture is possible when tangential stress reaches the critical value equal to local frictional force.

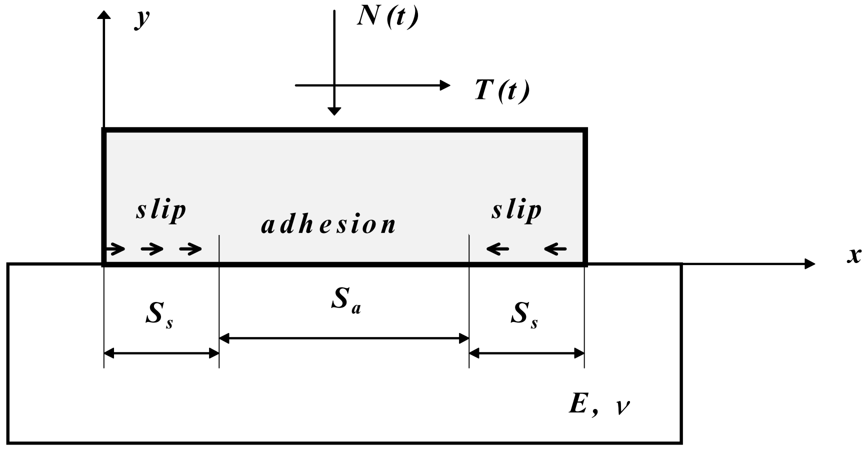


Fig. 1.1. One - interface scheme of interaction in granular medium

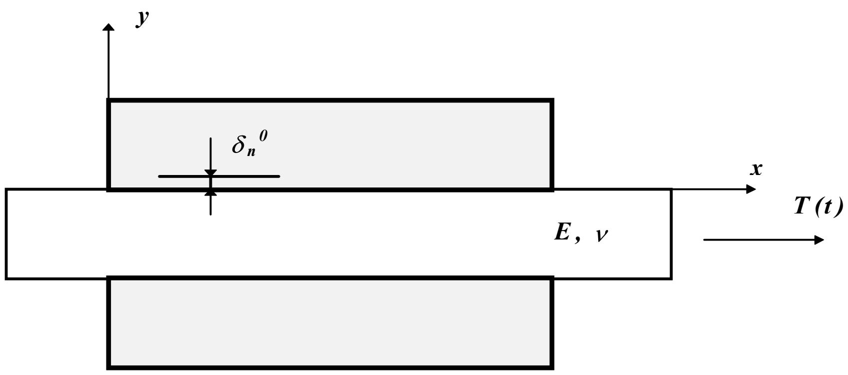


Fig. 1.2. Two - interface scheme of interaction in granular medium

The mathematical simulation of free boundaries creation and evolution may be based on the variational inequalities when the classic contact problem formulation is equivalent to the variational inequality solution or optimization problem with limitations in the form of inequalities

$$J(u) = \min_{w \in K,\ p \le 0,\ \tau \le f|p|} \max \max \left[0{,}5 \left(a(w, w) - L(w) + j(w) \right) \right]; \qquad (1)$$

where $a\,(w,w) = \int\limits_{\Omega} a_{ijkl}\ \varepsilon_{kl}(w)\ \varepsilon_{ij}(w)\ \mathrm{dw}$ is qua dratic, $L(w) = \int\limits_{S_F} F\,w\,\mathrm{ds}$ is linear, $j\,(w) = \int\limits_{S_c} \left[\, p\gamma\ (x,w)(w_T - u_T)\right] \mathrm{ds}$

is nonlinear components of the functional, respectively.

A like model for adhesion failure description has been presented in [4] using the physical analogy between static adhesion and dynamic frictional contacting. The boundary of failure zone depends on relation of adhesive and substrate strength, test conditions and other factors. Hence, the problem of free boundary with limitations in the form of inequalities $|\tau| \le \tau_s$, $|\tau| \le \tau_a$, $p \le p_a$ (where τ_s, τ_a, p_a are adhesive yield point, shear and gap adhesion strength, accordingly) is formulated. The nonlinear term of functional (1) describing the plastic flow of thin intermediate layer and damage of adhesive bonds for bilateral contact may be expressed as:

$$j^{t+dt}\left(w - u^{t+dt}\right) = \int\limits_{S_c} \left[\tau^{\mathrm{t}} + \left(\tau_s - \tau_a^t\right)\theta_0\left(\tau_a - \tau_s\right)\right]\left(\,|\,w_T - u_T\,|\,\right)\mathrm{ds}\,, \qquad (2)$$

with its relaxation and further partial rehabilitation according to the dependence

$$\tau_a^{t+dt} = \tau_a^t - k_\tau\,(t)\,\tau_a^t\,\theta_0\left\lfloor\left(\left|\,u_T^{t+\mathrm{dt}} - u_T^t\right|\right)/\,dt\,\right\rfloor, \qquad (3)$$

where $k_\tau\,(t)$ is relaxation coefficient and θ_0 is Heaviside function.

For the simulation of deformation process we use the procedure based on Udzava's algorithm with the steps of contact parameters correction in time.

The boundary element method with simple approximation partitioning of the surface region into linear segments have been used. The displacements at the nodes of the formed mesh are determined with the use of the Green's functions for the studied type of elastic counterbody (boundary element method).

The numerical results provide the conclusion that the creation and evolution of free internal boundary influence the deforming diagram of materials studied and may be used for elasitic and non-elastic properties prediction. For the mechanical analysis of supercompressible materials necessary to make it's structural unity (granular media, honeycomb structures, foams, etc.). The localization method of structural unity modelling mentioned above may be used for this goal.

2. The honeycomb structures

In realization of the super-compressibility effect anisotropy materials, which show negative Poisson's ratios in certain directions, are found to be interest. That is why the two-dimensional rod system (figure 2.1), which can be treated as a model of a porous material with regularly arranged honeycombs, has been examined. A similar model was described earlier [5]. In the present work, unlike that in [5], the rod elasticity has been accounted for and the relationship of Poisson's ratio to a number of parameters studied.

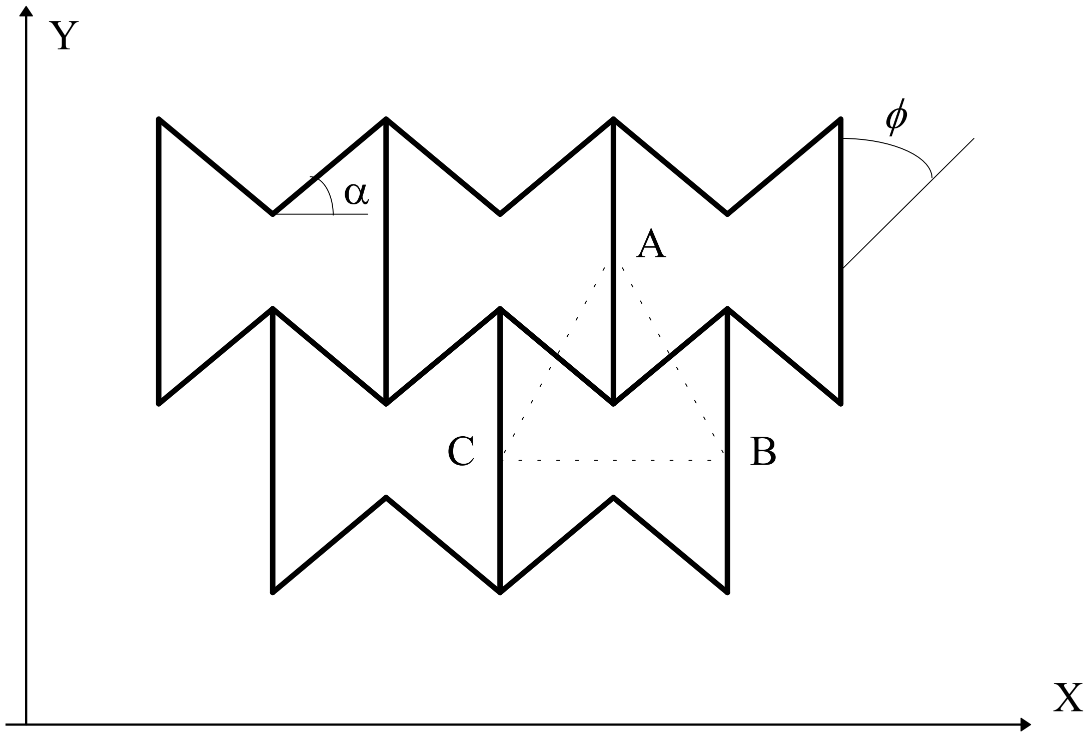


Fig. 2.1. Two-dimensional rod system

2.1. The method of modeling

We characterized the deformational behavior of the material by the behavior of a structural unit limited by a contour ABC (figure 2.1). Similarly to the method described in another work [6] for a cellular material, we took the given structural unit as a free body being in an equilibrium state (figure 2.2).

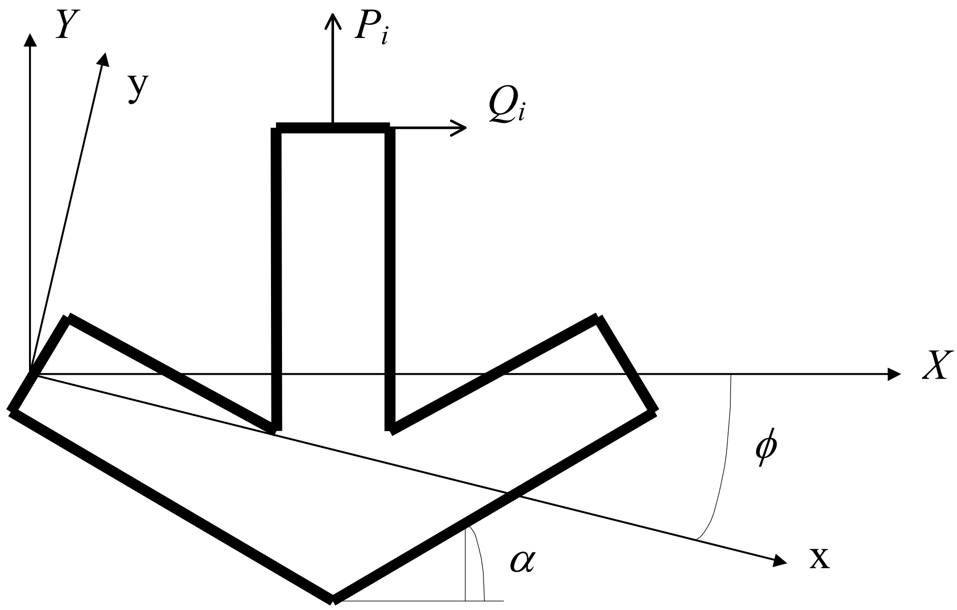


Fig. 2.2. Structural unit

We exchanged the bonds at the rod ends for the tensile P_i and bending Q_i forces. Under the action of these forces the points of their application got shifted in relation to the point where the rods were joined. We assumed the relationship between the strains and stresses to be defined with the help of two sets of variables which include relative displacements of the rod ends and the forces which led to those displacements. Having thus determined the tensor's components for elastic modules, it was possible to study the dependence of Poisson's ratio of this system ν on the loading angle ϕ, forming angle α, filler volume (volume fraction of the material of a rods) V_f and Young's modulus of the matrix material E_m.

Particularly, for the loading angles $\phi = 0$ and $\phi = \pi/2$ the expression for ν can be written as follows:

$$\nu(\phi = 0) = -\frac{(r - \sin(\alpha))\sin(\alpha)(\delta_{Q_2} - \delta_{P2})}{2\delta_{P1} + \delta_{Q_2}\cos^2(\alpha) + \delta_{P2}\sin^2(\alpha)}$$

$$\nu(\phi = \pi/2) = -\frac{\cos^2(\alpha)\sin(\alpha)(\delta_{Q_2} - \delta_{P2})}{(r - \sin(\alpha))(\delta_{Q_2}\sin^2(\alpha) + \delta_{P2}\cos^2(\alpha))}$$

Where $\delta_{Q_i}, \delta_{P_i}$ are the parameters having the meaning of rods' compliance found from the elasticity theory relationships and depending on the filler volume V_f, angle α and parameter r, characterizing the ratio of length of the vertical b and inclined a rods.

We assumed that for low values of V_f the rod's shape at bending loading could be found from the known equation of bending. This assumption allows the usage of the said method in accounting for the influence the matrix material on the deformation characteristics. In a such case, by using Winkler's hypothesis for finding the compliance δ_{Q_i}, the equation of bending must be solved for a distributed loading that is proportional to the transverse displacements of the rod. Winkler's rigidity factor of the base depends on the elastic properties of the matrix and the cell shape. In order to determine δ_P, the first order differential equation should be solved with taking into account the additional stress caused by adhesion between the matrix and rods.

2.2. Results

Figure 2.3 shows the relationship of ν to the loading angle ϕ for the following parameter values: $V_f = 0.2$, $r = 2$, $\alpha = 0.5$ *[rad]*, and Poisson's ratio of material of the rods $\nu_f = 0.1$. With $\phi \approx 0$ and $\phi \approx \pi/2$, Poisson's ratio of the model had negative values and there was a symmetry in the said relationship to the coordinate axes 0X and 0Y; this corresponds to the cell symmetry.

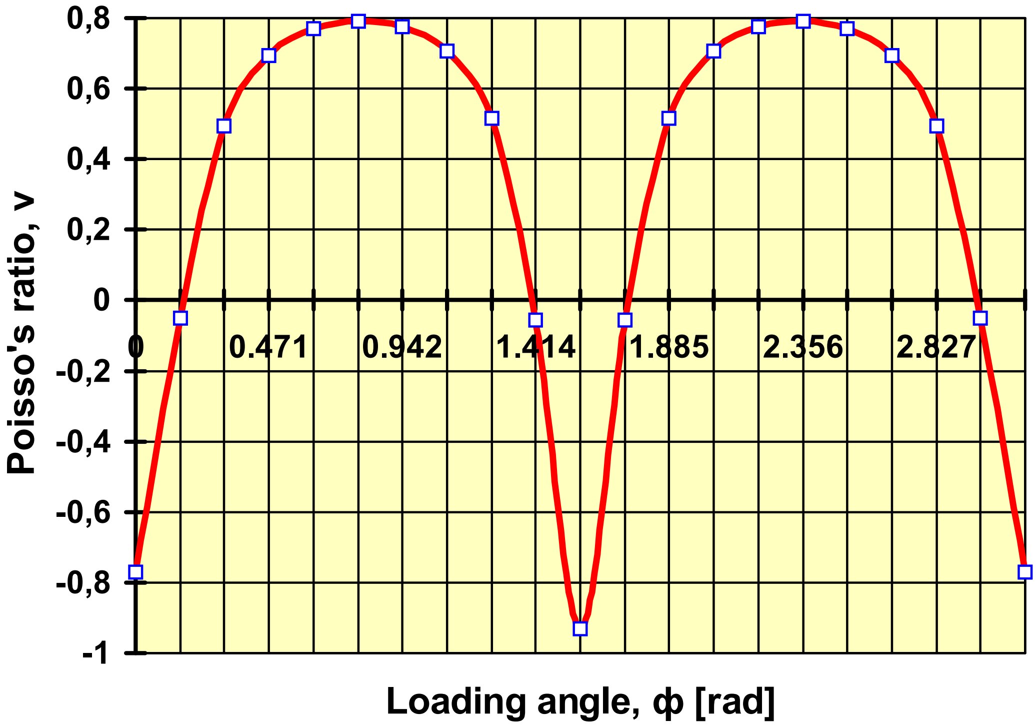


Fig. 2.3. Relationship of Poisson's ratio to the loading angle for $\boldsymbol{V_f = 0.2, r = 2, \alpha = 0.5}$ ***[rad]***, $\boldsymbol{\nu_f = 0.1}$

The analysis of the relationship ν on the α revealed that with angle $\phi = 0$, ν would sharply become lower if the α was growing, whereas with $\phi = \pi/2$ the Poisson's ratio would tend to zero at high values of α (figure 2.4). This could be explained by the fact that with increasing α, the degree of cell «concavity» increased, and the bending rigidity of the rods also increased (figure 2.2).

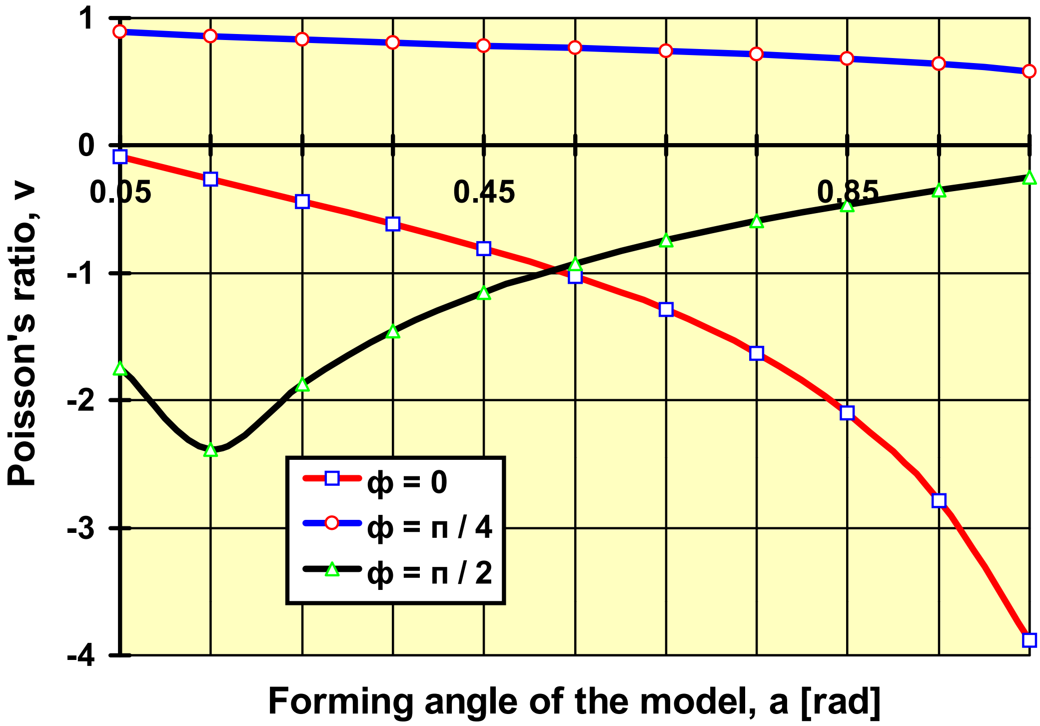


Fig. 2.4. Relationship of Poisson's ratio on the forming angle for $\boldsymbol{V_f=0.1, r=2, \nu_f=0.1}$

Figure 2.5 shows the relationship of Poisson's ratio to the filler volume V_f. The value can be seen to rise smoothly in the directions $\phi = 0$ and $\phi = \pi/2$ when V_f was increased. This can be explained by that on increasing V_f the rod compliance became lower; the bending compliance δ_Q would change at a higher rate than δ_P.

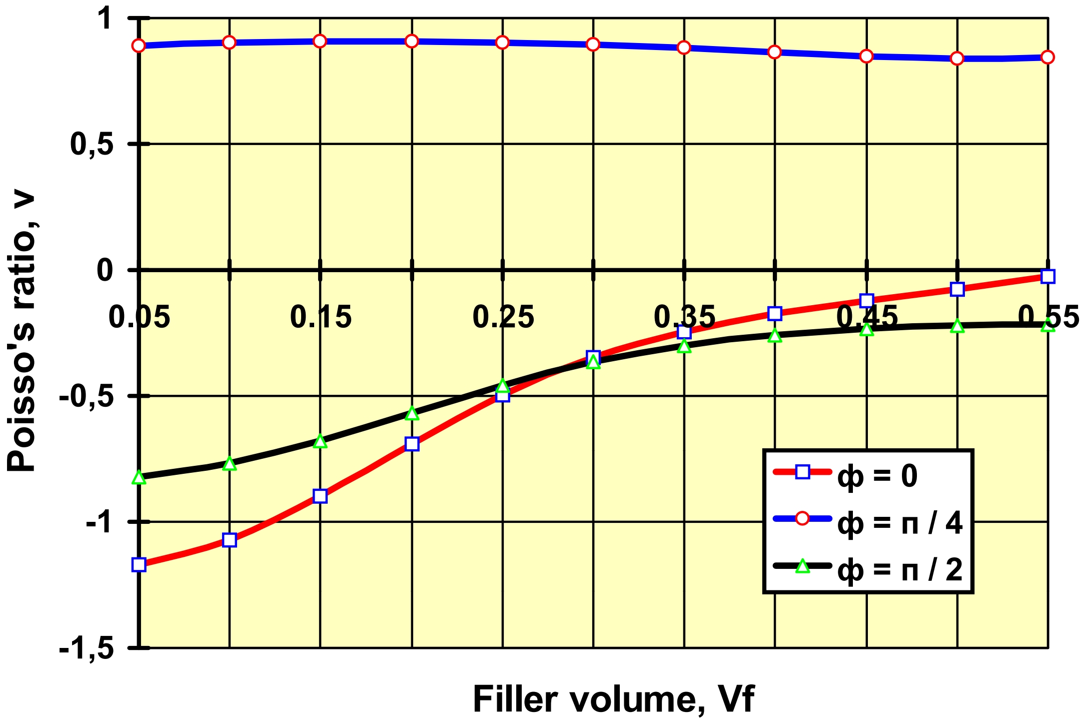


Fig. 2.5. Relationship of Poisson's ratio to the filler volume for ***r* = 3, *α* = 0.4 [rad], ν_f= 0.1**

The established relationship of characteristics $\delta_{Q_i}, \delta_{P_i}$ to Young's modulus for the matrix E_m was used to find a relationship of ν to E_m (figure 2.6) for the following parameter values: $\alpha = 0.5$ *[rad]*, $V_f = 0.2$, $r = 2$, $\nu_f = 0.1$ and Young's modulus of rods E_f*=15000 Pa*. The calculations made showed, that on increasing E_m Poisson's ratio would grow in the directions $\phi = 0$ and $\phi = \pi/2$, since the material of matrix resisted bending more strongly than it did tension.

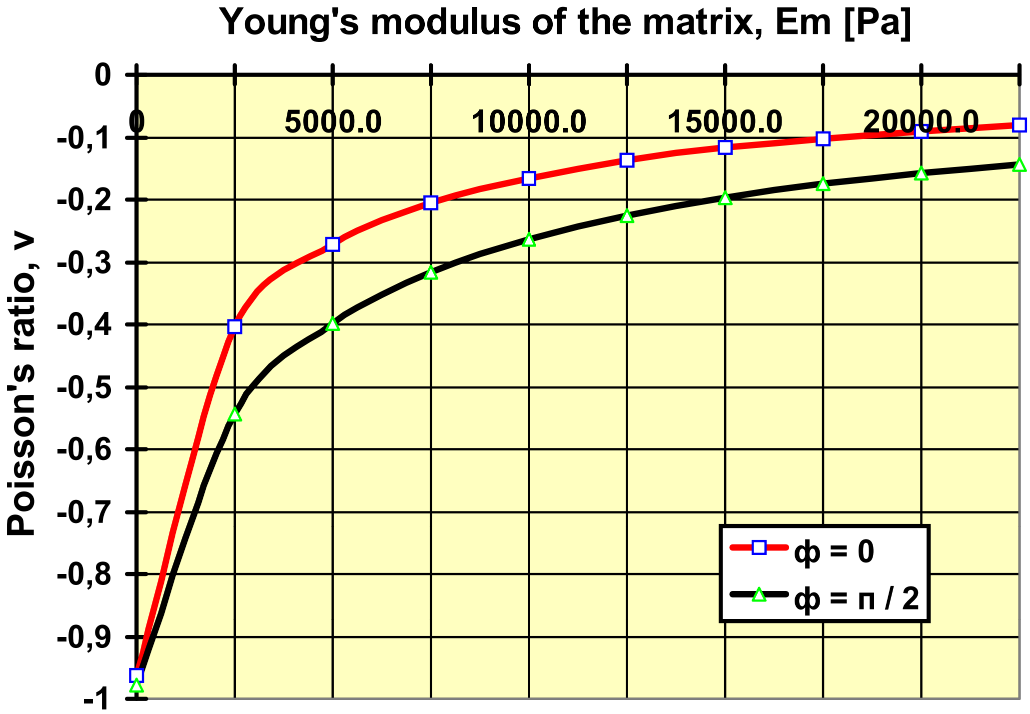


Fig. 2.6. Relationship of Poisson's ratio to Young's modulus of the matrix for α $= 0.5$ *[rad]*, $V_f = 0.2$, $r = 2$, $\nu_f = 0.1$, $E_f = 15$ *KPa*

2.3. Conclusion

The process of deformation was analyzed for an anisotropy super-compressible material by means of assuming a separate structural unit with rod-like elements. A linearly elastic state was assumed to determine the components of a two-dimensional tensor of elastic modulus of the material. The relationship of Poisson's ratio to the loading angle, filler volume and filler properties was examined. It was found out that the material in consideration showed a negative Poisson's ratio.

3. Foams

In modelling strained behaviour of foamed materials, a stressed cell system is usually considered. From the analysis viewpoint of the anomalous elasticity under question, the foamed polymeric materials with opened cells have been most investigated. The existig interpretations of the negative Poisson's ratios, observed in the course of said foam deformation, have been based on the possibility of inward motion of the walls of every cell [7]; also, a material structure was assumed as formed by telescopically folded rods [2]. However, the analysis of experimental fidings related to deformation of foams with closed cells, as well as of their models, had been described in literature inadeguately.

3.1. The physical model

The physical model of a closed-cell foam material, produced either by foaming a liguid substance or by microsphere compounding (in the case of syntactic foam), or by microsphere caking (gluing). In order to determine the elastic moduli, a fragment of this material can be represented as a set of bonded thin-walled spherical shell; the picture corresponds a real foam structure (figure 3.1).

To obtain a structure with ν<0, we have used the strong dependence of the bending rigidity of elastic rods, plates and shells on the initial curvature, according to the criterion, described elsewhere [8], in the form of tangential k_t to normal k_n rigidities relation, viz., $\frac{k_t}{k_n} < 0$. Thus, positive values of Poisson's ratio for celled materials having a common structure, energetically favourable for free foaming, result from the positive curvature of most cells. However, if the volumetric compression was large, a part of the cell surface acquires first zero then negative curvature.

We have conducted research and made an inference about the role of the cell-size-nonuniformity factor in the deformation of foams. The said nonuniformity was thought to result from different conditions of both the formation and development of cells in the bulk (figure 3.2). The cells of small size were found mostly to hold their original shape, but larger cells showed relatively low rigidity, and when compressed, would get deformed similarly to thin-walled shells, with a possibility of losing stability. Thus, the volumetric compression of a foamed material is mainly realised at the expense of decreased free volume of larger cells. Various modes of stability loss can take place depending on the loading regimes. Separation of cells according to deformation levels was found to cause anomalous elastic behavior in converted clozed-pore foams; the anomaly is characterized by negative values of Poisson's ratio.

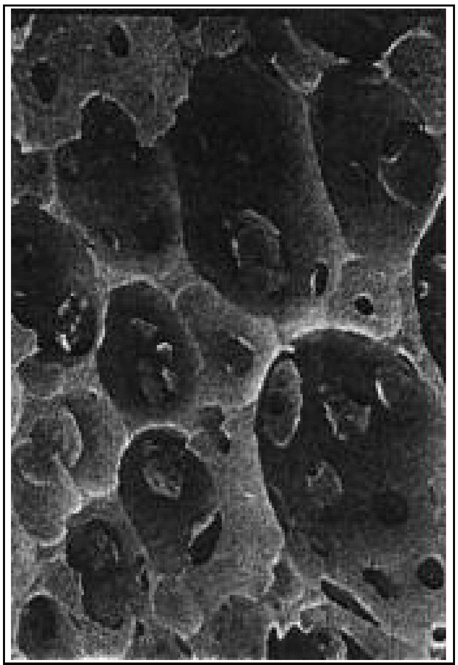


Fig. 3.1. Conventional silicone rubber foam [7]

It is worth mentioning, that the given deformation behavior reguired to reach a collapse, can be achieved as a result of cooperative deformation of the cell system, being symmetrical along three coordinate axes, without losing the cell continuity. The pronounced symmetry and presence of an adequate amount of particles appeared to increase the isotropy of the medium, as well as its producibility. In view of all this, it seems expedient to go from the statistical description of a medium to a local one on the scale of a structural unit.

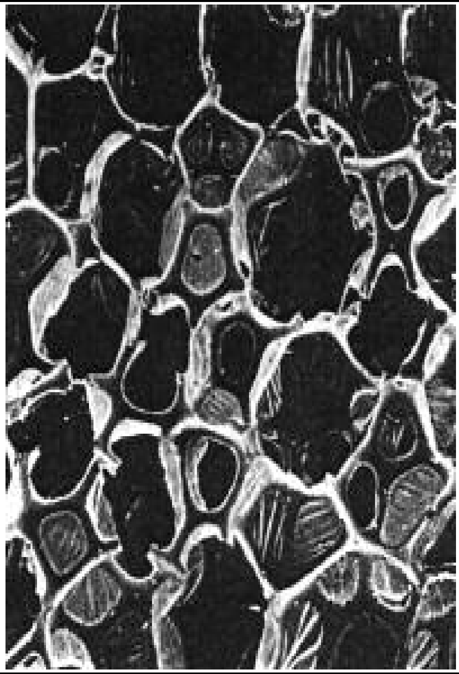


Fig. 3.2. Conventional polyester foam [7]

In view of the above, a structural model is suggested of an isotropic clozed-pore foamed material. The model is a system of stable (holding the original spherical shape) and convertible cells (figure 3.3). In order to examine the deformation effects, a structural unit is taken off composed of a central convertible cell and six satellite cells of a much smaller size, arranged symmetrcally.

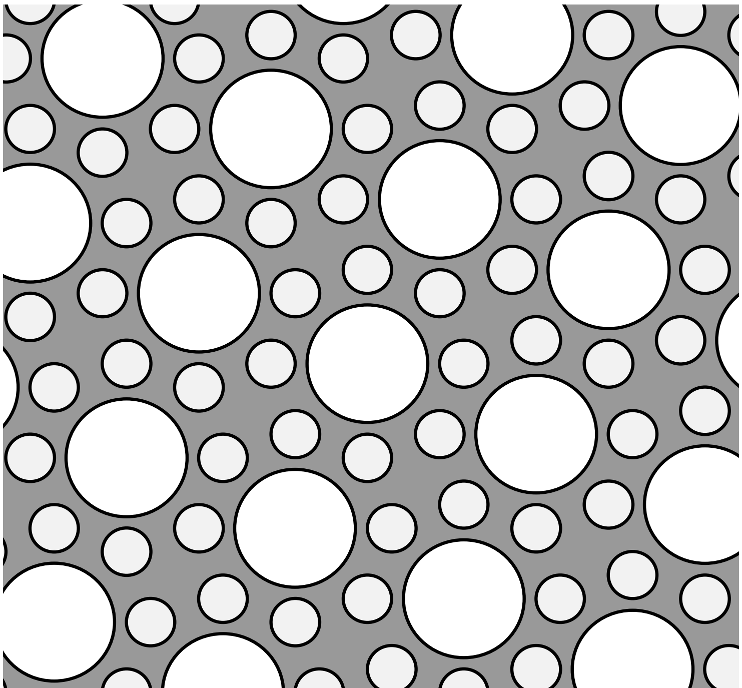


Fig. 3.3. The model is a system of stable and convertible cells

This model of a material can be realized by foaming a thermoplastic polymer, with making use of a pore former having fractions of two sizes. The model can also be realized by producing a foam by way of regularly arranging spherical cells of various sizes. A structural unit of a such material provides for a possibility of its further conversion by means of all-round compression of the initial mass; on a certain stage, this treatment can by combined with a thermal treatment at temperatures within the softening range, and cooling down to the room temperature with the purpose of retaining the obtained converted structure (as shown in figure 3.4), maintaining the isotropy of the elastic properties.

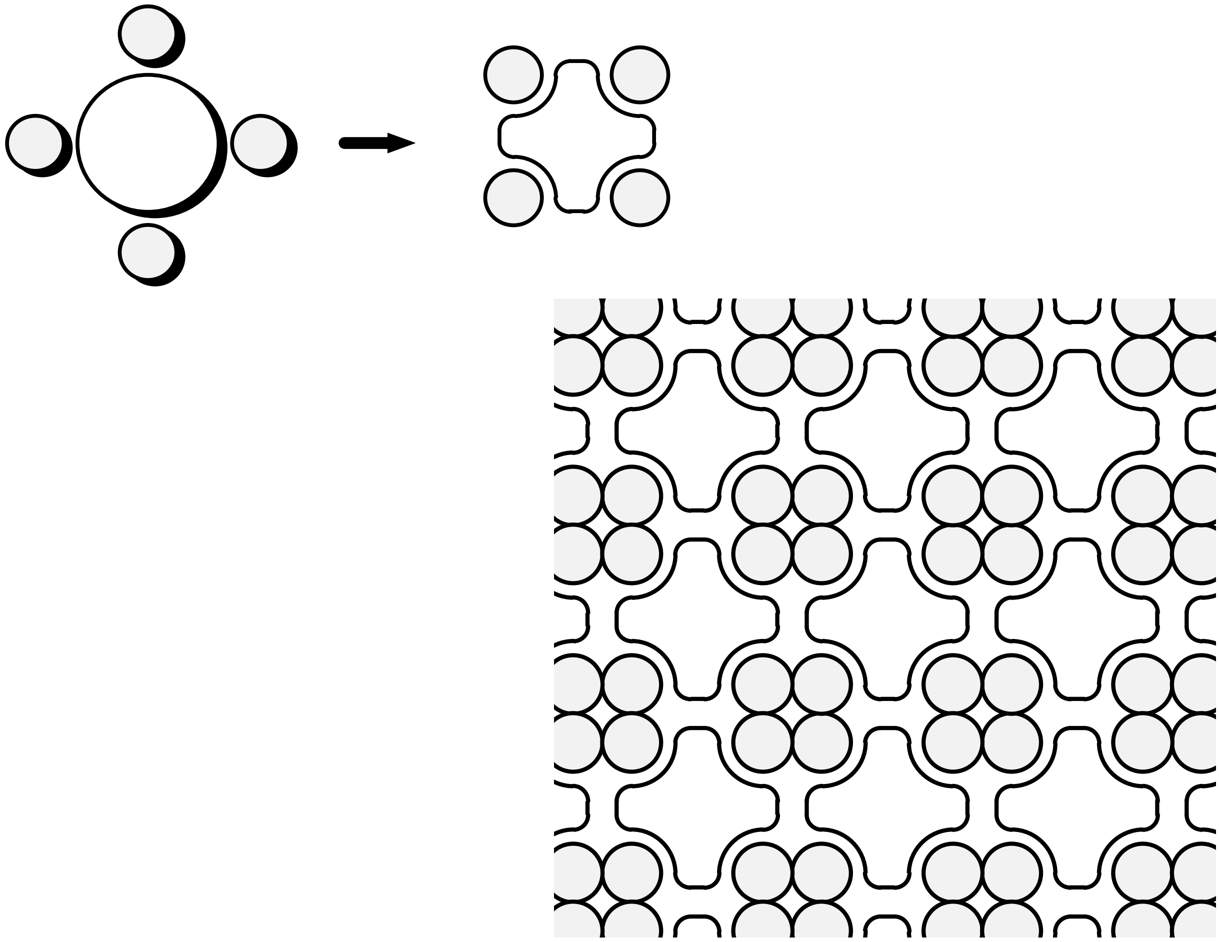


Fig. 3.4. The model is a system of stable and re-entrant cells

The shear rigidity of a such system depends on the deformation characteristics of the original material, whereas the normal rigidity depends on the convertibility of the cells at the expense of the free volume. The structural unit of a converted foam, formed in this manner, can possess the following features:

- low normal rigidity as compared with the shear rigidity, thus satisfying the criterion of high shear and low normal rigidities;

- prior conversion of the central cell owing to its relatively low rigidity towards the satellites.

3.2. Computer test

In order to examine the deformation behaviour of foams by numerical methods of mechanics, the finite-element approximation of a fragment of an elastic heterogeneous medium was applied [9].

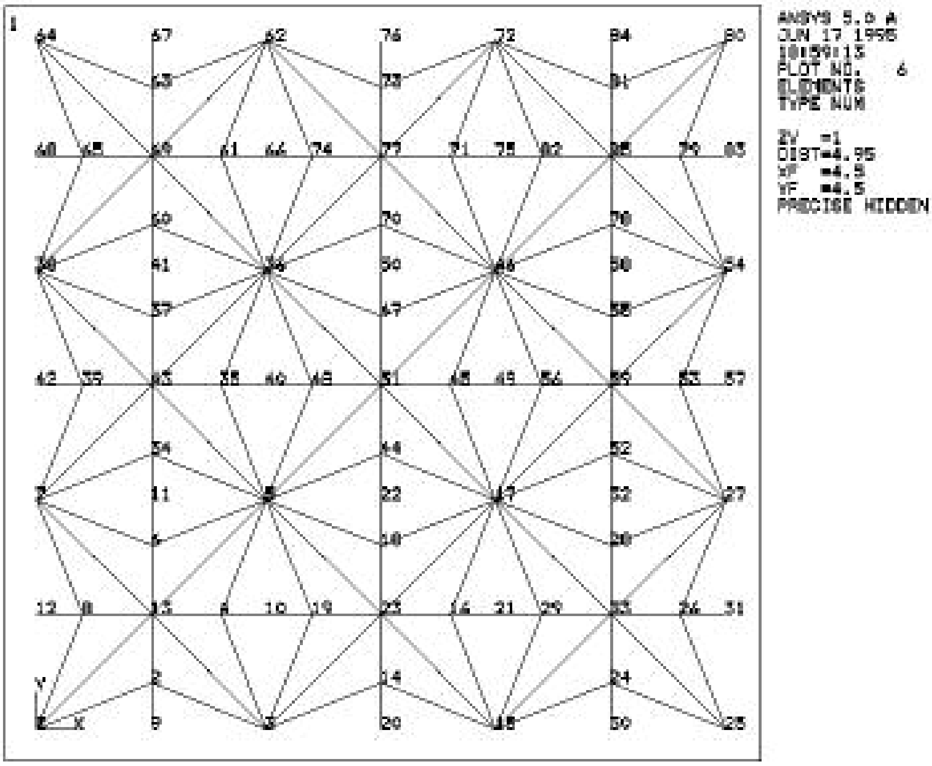


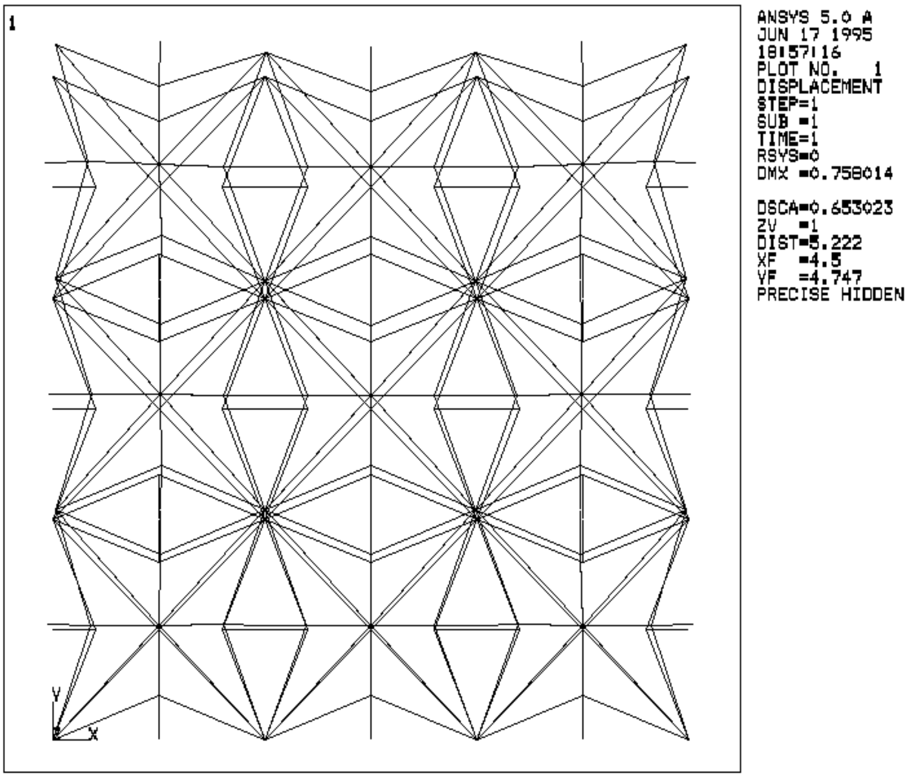
Fig. 3.5. The discretization of regular cell structure fragment

Every structural unit was constructed and joined to neighbouring units by means of eight triangular plate elemtnts simulating the characteristics of typical subregions of the materials, eight rod elements arranged along the profile and characterizing the reinforcement of the cell boundaries, and four rod elements to connect the edge midpoints of the neighbouring structural units (figure 3.5).

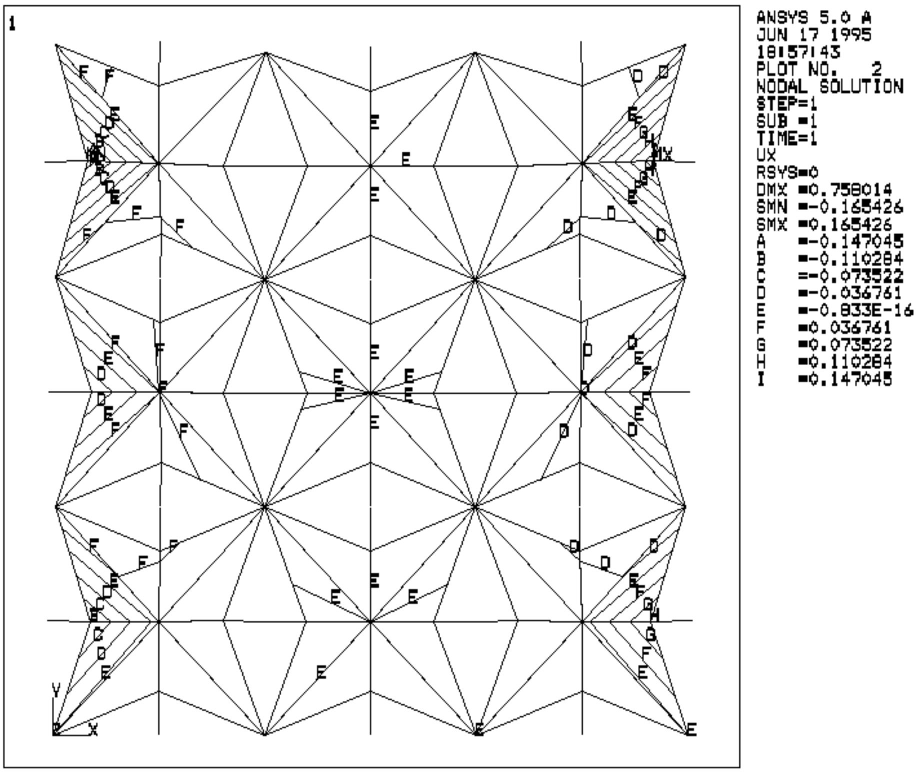
By varying the ratio of Young's moduli of the plate elements to rod elements, one has a possibility to treat different heterogeneous materials, foams included, as a limiting case of existing a phase with a zero elastic modulus.

The described model is equivalent to the suggested physical one since it describes variations in the shape of cells at the expense of free volume, if structural units are bonded to provide for the reguired deformation mode.

The method of finite element (software ANSYS [10]) was used to calculate the displacement fields where a fragment of the converted structure with concaved cells was stretched uniaxially. The calculation for detailing 3×3 and 5×5 structural units showed, that trasverse displacement of the nodal points belonging to the right end of the fragment become positive, i. e. the model gives negative values of Poisson's ratio (figure 3.6).



a



b

Fig. 3.6. Transverse displacement of re-entraned celled structure:

a - before and under uniaxial tensile loding;

b - the levels of displasement

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